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DARK MATTER PROPERTIES IN GALAXY U5750

Abstract. We investigate the properties of dark matter (DM) distribution in spiral galaxy U5750, employing the well known and widely used phenomenological density profiles such as pseudo-isothermal, Burkert, Navarro-Frenk-White, Einasto, Moore and exponential sphere. For simplicity we assume that DM distribution is spherically symmetric without accounting for the complex internal structure of the galaxy. We fit the rotation curve observational data of galaxy U5750 for each profile. We infer the model free parameters and estimate the total DM mass, and compare them with those reported in the literature. To discriminate the best fit profile among the considered ones, we make use of the Bayesian Information Criterion (BIC). On the basis of the performed statistical analysis, we provide physical interpretations for choosing certain profiles. In addition, by assuming that DM possesses non-zero pressure, we solve the Newtonian hydrostatic equilibrium equation and construct the pressure profiles as a function of the radial coordinate for each above mentioned profile. Combining the density profiles with the pressure profiles we obtain equations of state for the DM in the considered galaxy. Further, we calculate the speed of sound in the DM medium and show that it behaves not unequivocally for the adopted profiles, though it decreases with an increasing DM density. Finally, we calculate the refracting index and discuss about astrophysical implications of the obtained results.

Keywords: dark matter, rotation curves, equation of state, the speed of sound, the refractive index.

I. Introduction

At the beginning of the last century astronomers discovered that some stars and galaxies behave differently than theory predicted. The rotation of more distant parts of galaxies did not obey the laws of celestial mechanics. This marked the beginning of a new era to search for DM. The modern epoch of the dark (latent) mass concept began at the end of the last century in connection with the need to explain the kinematics of gas in massive galaxies, i.e. extended rotation curves (RCs) of the gaseous component that did not decrease with increasing distance even outside the stellar disk of galaxies [1,2]. The need to introduce additional mass also arose when simulating the mass distribution in elliptical and dwarf spheroidal galaxies, where disks were missing.

The problem of DM is one of the fundamental and yet unsolved problems in modern astrophysics and cosmology. Unlike ordinary visible/baryonic matter, DM does not participate in electromagnetic interactions, but manifests itself only through gravity (and possibly via weak interaction as well) and therefore is inaccessible to direct observations. It is believed that at present accounts for about 26.8 % of the total mass of all forms of matter in the Universe [3]. The true nature of DM still needs to be understood. It is assumed that DM consists of some experimentally undiscovered particles [4, 5]. They cannot be baryons, since in that case the cosmic microwave background and the large-scale structure of the Universe would be radically different. Nevertheless, up to date, a huge amount of information has been obtained, which allows one to judge about the distribution of DM and its role in the Universe. The number

of scientific works related to the problem of DM is steadily growing from year to year, which reflects a noticeable interest in this intriguing and at the same time fascinating topic [6–8].

In this paper we investigate the observational properties of DM in the spiral galaxy U5750, exploiting various phenomenological density profiles. By analysing the RC data points we infer all the model parameters. We suppose that the DM equation of state (EoS) in a particular galaxy, by default, must not depend upon the density profiles. Hence we intend to check whether this assumption is valid or not. Therefore we involve the Newtonian gravity for simplicity and solve the hydrostatic equilibrium equation to get the pressure profiles. By expressing the pressure in terms of the density for each individual model one derives the EoS of DM in the spiral galaxy U5750. Thus, the main objective of this work is to test whether the DM EoS depends on the adopted density profile or not. In addition we calculate the speed of sound and refractive index in the DM medium [9] as these features of DM can be crucial in structure formations and gravitational lensing effects.

The paper is organized as follows. In section II, we review the main density profiles which are used to study DM distribution in galaxies. In section III we describe methods involved in the paper to perform best fit analyses and derive EoS of DM. In section IV the major results of the paper are shown, namely, the observational data of U5750 RC is analysed. The derivation of the EoS of DM in spiral galaxy U5750, the speed of sound and the refractive index are presented and discussed. In section V our conclusion is summarized.

II. Dark matter profiles

The distribution of DM in the halos of galaxies is not uniform, concentrating at their centers and dropping off to the periphery. The corresponding distribution function of DM or its profile is usually found by the methods of numerical modeling of the dynamics of stars in galaxies. In this work we have selected the most famous and commonly applied DM density profiles such as pseudo-isothermal (ISO), Moore, Burkert, Navarro-Frenk-White (NFW), Einasto and exponential sphere. All, these profiles possess two model parameters: the DM density at galactic centers or characteristic density ρ_0 and the scale radius r_0 , apart from the Einasto profile, which has another extra free parameter. In Ref. [10] the model parameters of the above listed profiles were estimated for some galaxies. Inferring the numerical values of these parameters is a standard task that can be solved by studying the dynamics of galaxies. Here for our analyses we involve the following models:

- ISO profile [11]:

$$\rho_{Iso}(\xi) = \frac{\rho_0}{1 + \xi^2}, \quad (1)$$

where $\xi = r/r_0$ is the dimensionless radial coordinate/distance.

- Exponential sphere [12]:

$$\rho_{Exp}(\xi) = \rho_0 e^{-\xi}, \quad (2)$$

- Burkert profile [13]:

$$\rho_{Bur}(\xi) = \frac{\rho_0}{(1 + \xi)(1 + \xi^2)}, \quad (3)$$

- NFW profile [14], which is proposed based on cosmological models of halo formation:

$$\rho_{NFW}(\xi) = \frac{\rho_0}{(1 + \xi)^2}, \quad (4)$$

- Moore profile [15]:

$$\rho_{Moo}(\xi) = \rho_0 \xi^{-1.16} (1 + \xi)^{-1.84}, \quad (5)$$

- Einasto profile [16]:

$$\rho_{Ein}(\xi) = \rho_0 \exp\left[2n\left(1 - \xi^{1/n}\right)\right], \quad (6)$$

where α is the Einasto free parameter.

The dependence of ρ/ρ_0 on r/r_0 for different DM halo profiles is illustrated in Fig.1 (left panel). At large distances the density always goes down.

III. Methods

In the RCs of galaxies the rotation (linear) speed v of stars and gas depends on the distance r from the center to the halo. The RC allows to determine the galaxy's mass profile as a function of the radial distance. By equating the centrifugal and gravitational forces acting on a star moving in a circular orbit at a distance r from the center of the galaxy one finds the circular velocity

$$v(r) = \sqrt{\frac{GM(r)}{r}}. \quad (7)$$

The RCs do not drop off according to Newton's laws but stay flat near the edge of the galaxies. The DM mass $M(r)$ in Eq. (7) can be computed by integrating the mass balance formula (see Eq. (10) below)

$$M(r) = \int_0^r 4\pi r'^2 \rho(r') dr', \quad (8)$$

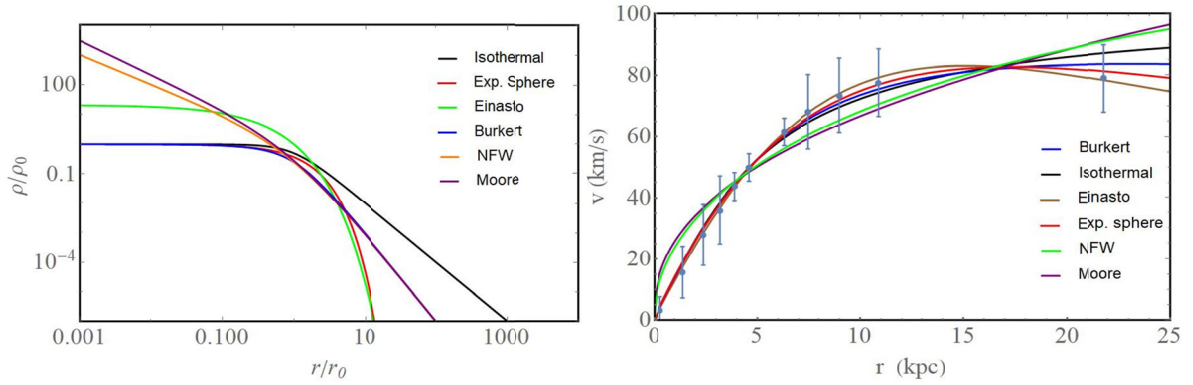


Figure 1 - Color online. Left panel: different phenomenological DM density profiles. We choose $\alpha = 1.5$ for the Einasto profile. Right panel: RCs of galaxy U5750 and fits obtained by using the DM profiles considered in this work

where $\rho(r)$ is the DM density profile taken from Eqs. (1) – (6).

In order to search for the best fit parameters of each DM profile, we use the χ^2 function which is defined as follows

$$\chi^2 = \sum_{i=1}^N \left[\frac{v_i - v(\rho_0, r_0, r)}{\sigma_{v,i}} \right]^2, \quad (9)$$

where v_i and $\sigma_{v,i}$ are the N data points from RC of galaxy U5750 and their corresponding errors, respectively (see Fig. 1, right panel). The RCs for each DM profiles are described by the $v(\rho_0, r_0, r)$, given by Eq. (7). We apply the Levenberg-Marquardt method (This algorithm is an iterative technique that locates the minimum of a function that is expressed as the sum of squares of nonlinear functions. It consists in a combination of the Gauss–Newton algorithm and the method of the steepest descent gradient) [17, 18] to minimize Eq. (9).

To derive an EoS of DM in the considered galaxy we perform computations in the Newtonian gravity (NG) for simplicity and clarity. So we will proceed with the Newtonian hydrostatic equilibrium equations:

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r), \quad (10)$$

$$\frac{dP(r)}{dr} = -\rho(r) \frac{GM(r)}{r^2}, \quad (11)$$

$$\frac{d\Phi(r)}{dr} = \frac{GM(r)}{r^2}. \quad (12)$$

where $M(r)$ is the mass profile enclosed inside a sphere of radius r , $P(r)$ is the pressure profile, G is the gravitational constant, $\Phi(r)$ is the internal gravitational potential of the DM distribution.

IV. Results and discussion

In this section we present our main results providing technical details and discussions.

A. Best fit models

By exploiting each DM profile we infer the model free parameters from the fit and construct theoretical RCs which become flat at large distances. The best fit parameters are listed in table I and shown in figure 1 (right panel). The χ^2 values are also shown in the last column of table I. For the NFW profile the parameters are unconstrained and are not listed in Table I. This is in line with the results in Refs. [19, 20], where the large uncertainties in the NFW parameters have been found. This is mainly caused by the linear rise of the inner rotation curve. The same is true for the Moore profile, whose fit was not good.

To compare the 4 profiles of table I, which have different number of parameters and are not nested into each other, we employ the Bayesian Information Criterion (*BIC*) (The BIC is a selection criterion among a finite set of models, conceived to solve the overfitting issue when increasing the number of parameters in the fitting function) [21]. From the χ^2 definition, the BIC is defined as

$$BIC = \chi^2 + k \ln N, \quad (13)$$

where k is the number of model parameters. For example, in the case of the Einasto profile $k = 3$, while for all the other profiles $k = 2$. A profile with a minimum BIC value is favored, according to Ref. [22]. As one can see from Table I for galaxy U5750 the Einasto profile has the minimum BIC, while the ISO profile has the maximum value.

Table I - Best fit model parameters for galaxy U5750.

For comparison, the mass of the dark matter in this galaxy is $2.9 \times 10^{10} M_{\odot}$ according to Ref. [23]

Profiles	$\rho_0 \pm \sigma_{\rho_0} [10^{-3} M_{\odot} / pc^3]$	$r_0 \pm \sigma_{r_0} [kpc]$	$M \pm \sigma_M [M_{\odot}]^a, M_{\odot}$	$M \pm \sigma_M [M_{\odot}]^b, M_{\odot}$	BIC	χ^2
Burkert	11.73 ± 0.65	7.16 ± 0.38	$(3.54 \pm 0.42) \times 10^{10}$	$(6.88 \pm 1.61) \times 10^9$	54	0.70
ISO	10.71 ± 1.00	4.23 ± 0.38	$(3.85 \pm 0.69) \times 10^{10}$	$(2.19 \pm 0.87) \times 10^9$	61	1.37
Einasto ^c	2.14 ± 0.10	9.37 ± 0.20	$(3.11 \pm 0.23) \times 10^{10}$	$(1.24 \pm 0.13) \times 10^{10}$	37	0.13
Exp. sphere	12.50 ± 0.50	5.11 ± 0.19	$(3.34 \pm 0.32) \times 10^{10}$	$(3.36 \pm 0.55) \times 10^9$	49	0.47

^aThe DM total mass is calculated using the last RC data point in the halo for r

^bThe DM total mass is calculated using the scale radius r_0

^cFor the Einasto profile free parameter $\alpha = 0.73 \pm 0.05$

In figure 1 (right panel) the gray thick points show the observational data points of galaxy U5750 with the attached error bars. The DM profiles are shown with solid curves: ISO (black), exponential sphere (red), Einasto (brown) and Burkert (blue), NFW (green), Moore (purple).

B. Equation of state

In order to obtain the formulas of the pressure in NG, the density profiles Eqs. (1) - (6) are plugged in Eqs. (10) - (11) and then are integrated to fulfill boundary conditions i.e. at infinity the pressure must be zero. However, due to the complexity for the Einasto, Moore, NFW and Burkert profiles all computations

are carried out numerically, while for the ISO and exponential sphere profiles the sought expressions can be derived analytically. As a result, the pressure formulas of the DM halo for the ISO and exponential sphere profiles in Eqs. (1) and (2) are give by, respectively

$$P(\xi) = 2G\pi r_0^2 \rho_0^2 \left[\frac{\pi^2}{4} - \frac{2}{\xi} \arctan(\xi) - \arctan(\xi)^2 \right], \quad (14)$$

$$P(\xi) = 2G\pi r_0^2 \rho_0^2 \left[4\Gamma[-1, \xi] - 8\Gamma[-1, 2\xi] - 4\Gamma[0, 2\xi] - e^{-2\xi} \right], \quad (15)$$

where the gamma function is defined as $\Gamma(a, z) = \int_z^\infty e^{-t} t^{a-1} dt$, with $a = 0, -1$

Combining Eqs. (14) and (15) together with Eqs. (1) and (2), we obtain the EoS for the ISO and exponential sphere, respectively

$$P(\rho) = 2G\pi r_0^2 \rho_0^2 \left[4\Gamma\left[-1, \ln\left(\frac{\rho_0}{\rho}\right)\right] - 8\Gamma\left[-1, 2\ln\left(\frac{\rho_0}{\rho}\right)\right] - 4\Gamma\left[0, \ln\left(\frac{\rho_0}{\rho}\right)\right] - \frac{\rho^2}{\rho_0^2} \right], \quad (16)$$

$$P(\rho) = \frac{G\pi r_0^2 \rho_0^2}{2} \left[\pi^2 - \frac{8\sqrt{\rho}}{\sqrt{\rho_0 - \rho}} \arctan\left[\frac{\sqrt{\rho_0 - \rho}}{\sqrt{\rho}}\right] - 4 \arctan\left[\frac{\sqrt{\rho_0 - \rho}}{\sqrt{\rho}}\right]^2 \right], \quad (17)$$

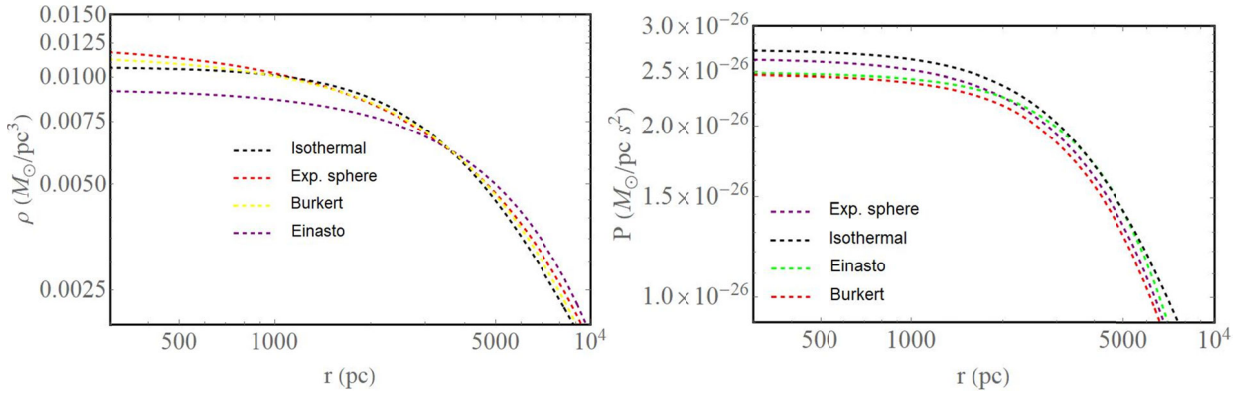


Figure 2 - Color online. Logarithmic density profiles (left panel) and logarithmic pressure profiles (right panel) of DM in the halo for the profiles listed in table I

In figure 2 we plotted $\rho(r)$ (left panel) and $P(r)$ (right panel) for galaxy U5750, using Eqs. (1) – (6) and (14) – (15). As one can see, the pressure profiles follow the same trend as the density profile. In figure 3 we plot the equation of state (left panel), obtained from Eqs. (16) and (17) for ISO and exponential sphere profiles and from numerical integrations for the Burkert and Einasto profiles.

It should be noted that for the ISO, exponential sphere and Burkert profiles ρ_0 is the central density. However for the Einasto, NFW and Moore profiles ρ_0 is the characteristic density, i.e. the density near the galactic center can be chosen arbitrarily larger than ρ_0 in these models. Thus, as one can see from figure 3, the behaviour of the EoS is different for each profile. These results are similar to the ones obtain in Ref. [24].

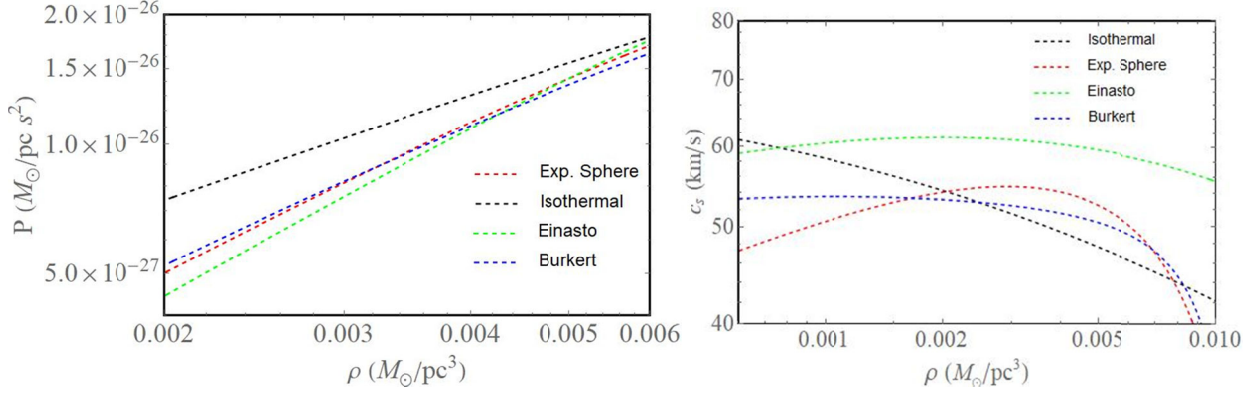


Figure 3 - Color online. The equation of state of DM (left panel) and speed of sound (right panel) in DM for the profiles listed in table I

C. Speed of sound

Perturbations due to DM, in adiabatic regime, can be accounted by means of the speed of sound

$$c_s^2 = \frac{\partial P}{\partial \rho}, \quad (18)$$

which can be deduced analytically from Eqs. (16) and (17) for the ISO and exponential sphere profiles, respectively,

$$c_s^2 = \frac{4G\pi r_0^2 \left[2(\rho_0 - \rho) - \rho \ln\left(\frac{\rho_0}{\rho}\right) - \left(2 + \rho \ln\left(\frac{\rho_0}{\rho}\right)\right) \right]}{\ln^2\left(\frac{\rho_0}{\rho}\right)}, \quad (19)$$

$$c_s^2 = \frac{4G\pi r_0^2 \rho_0^2 \left[(\rho_0 - \rho) - \sqrt{\rho(\rho_0 - \rho)} \arctan\left(\sqrt{\frac{\rho_0 - \rho}{\rho}}\right) \right]}{(\rho_0 - \rho)^2}, \quad (20)$$

In figure 3 (right panel) we plot the speed of sound, according to Eqs. (19) and (20) for the ISO and exponential sphere profiles, respectively and from numerical integration for the Burkert and Einasto profiles. For large densities the speed of sound goes down. This behaviour is utterly opposite of compact objects, where the speed of sound goes up with increasing density. However, in cosmology if the speed of sound is less than in its surroundings then this fact allows to form structures as galaxies, i.e. galaxies are formed inside dark matter halos.

D. Refractive index

It is also interesting to study the refractive index n induced by the DM halo of galaxy U5750. In the weak field regime it is given by

$$n(r) = 1 - \frac{\Phi(r)}{c^2} - \int \frac{GM(r)}{c^2 r^2} dr, \quad (21)$$

In figure 4 we plot the refractive index n as a function of the radial coordinate in the DM distribution for the above considered profiles in galaxy U5750. As expected, the value of the index is very small in the halo region. In the core region it grows slowly and becomes constant, slightly larger than in vacuum. So, the gravitational lensing is quite weak. However, with our current techniques it is possible to measure the lensing effects with high precision [25].

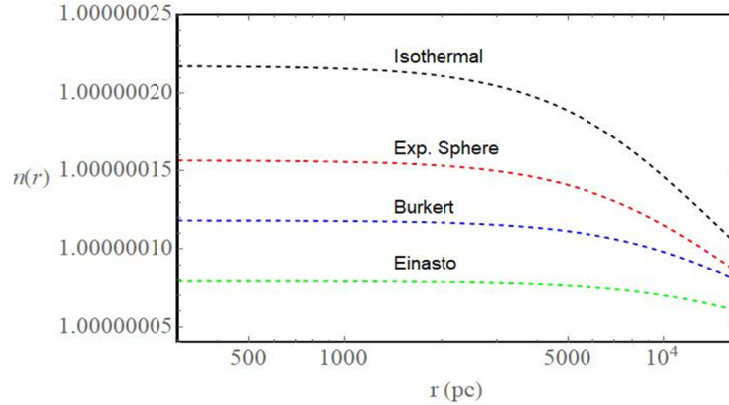


Figure 4 - Color online. The refractive index for DM in galaxy U5750 for the profiles listed in table I

V. Conclusion

We analysed the RC of the spiral galaxy U5750 and inferred the free parameters of the considered profiles with the help of the least square method. We used the well-known DM density profiles in the literature and, in addition, we considered as an instructive example the exponential sphere profile for comparison. Unlike other profiles, the exponential sphere profile is usually applied to study the inner parts of galaxies [12], and here we used it to the halo. Although this model has one of the lowest BIC and good estimation of the DM total mass, its use in the description of the halo is not physically motivated.

We considered also the NFW and Moore profiles for completeness, though the density profile diverges as $r \rightarrow 0$ causing the cuspy halo problem. Moreover, from the fit the parameters of these two models were not well constrained in accordance with Refs. [19, 20]. Indeed, we conclude that the two profiles are not appropriate to study DM EoS.

It turned out that only three profiles, i.e. ISO, Burkert and Einasto profiles, are suitable for the analysis of the DM equation of state as $r \rightarrow 0$, where the density tends to be finite avoiding cusps. Out of the three profiles, Einasto profile has the minimum BIC number and ISO profile the maximum one. The same is true for χ^2 values, though the Burkert and ISO profiles values are closer to unity. The difference in BIC between Einasto (with three free parameters) and ISO (with two free parameters) profiles is $\Delta BIC = 24$; the difference in BIC between Einasto and Burkert (with two free parameters) profiles is $\Delta BIC = 17$. These results exhibit a very strong evidence against the ISO and Burkert models. So, we can conclude that Einasto profile is the most suited profile to study the DM equation of state of the galaxy U5750.

Although the behavior of $\rho(r)$ and $P(r)$ profiles is similar, the equation of state $P(\rho)$ is not unique, we expected that it would be model independent. However, our supposition was not confirmed. This only means that DM problem remains not fully solved and understood. Therefore, one should propose new ideas, theoretical models and probably some experiments to better comprehend the nature of DM.

In addition we estimated the speed of sound and showed that it is different among the considered profiles. Nevertheless, the speed of sound had a tendency to decrease with increasing density for all profiles. This is a unique feature for DM which plays a key role in the large-scale structure formation.

Furthermore, we calculated the refractive index in the dark matter distribution. It was constant near the galactic core and was decreasing in the halo region. Its value was slightly larger in the galaxy filled with DM than in vacuum. This characteristic can be used to measure and explore gravitational lensing effects.

It would be interesting to study dark matter properties in other galaxies including their complex structures such as a core, bulge, disk and halo. This will be the issue of future studies.

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U5750 ГАЛАКТИКАСЫНДАҒЫ ҚАРАҢҒЫ МАТЕРИЯНЫҢ ҚАСИЕТТЕРІ

Аннотация. Қараңғы материя (ҚМ) мәселесі - қазіргі астрофизиканың, космологияның және элементар бөлшектер физикасының іргелі және әлі шешілмеген мәселелерінің бірі. ҚМ-ның бариондық материядан айырмашылығы, ол электромагниттік өзара әсерлесуге қатыспайды, тек гравитациялық әсерлесулер арқылы байқалады (мүмкін, әлсіз әсерлесу арқылы да), сондықтан оны тікелей бақылаулар арқылы тіркеу және зерттеу мүмкіндігі осы күнге дейін болмай жатыр. Қазіргі кезде ҚМ Әлемнің жалпы энергетикалық үлесінің 26.8% құрайтын материяның ерекше түрі. Астрономиялық бақылаулар қараңғы материя негізінен галактика іспетті ірі масштабты ғарыш объектілерінің және олардың кластерлерінің айналасында шоғырланатынын көрсетеді. Бұл ретте қараңғы материя галактика массасының 90% - на дейін құрайтын галодан тұрады.

Бұл жұмыста U5750 шиыршықты галактикасында ҚМ таралуы зерттеледі. U5750 галактиканың сипаттамаларын зерттеу үшін изотермиялық, Буркерт, Наварро-Франк-Уайт, Эйнасто және Мур сияқты белгілі және кеңінен қолданылатын тығыздық профилдері қарастырылды. Сонымен қатар, салыстыру үшін экспоненциалды сфераның профилі зерттелді. Әдетте ол галактикалардың ішкі бөліктерін зерттеу үшін қолданылады, бірақ мақалада мысал ретінде келтірілді. Галактика динамикасындағы қараңғы материяны есепке алу Ньютондық жуықтау аясында да сәтті жүзеге асырылады. U5750 галактикасы үшін изотермиялық профиль ең үлкен ВІС және ең үлкен χ^2 көрсетті. НФУ және Мур профилдері $r \rightarrow 0$ болған кезде, орталық тығыздық шексіздікке ұмтылады. Сондықтан бұл екі профиль ҚМ күй теңдеуін зерттеуге жарамайды, бірақ үлкен қашықтықта олар жазық айналу қисықтарын береді. Ал, Буркерт, Эйнасто және изотермиялық профилдері $r \rightarrow 0$ кезінде ҚМ күй теңдеуін талдауға өте қолайлы. Жоғарыда айтылған барлық профилдердің ішінде Эйнасто профилі үшін ВІС-тің мәні минималды, ал изотермиялық профиль үшін ВІС – тің максималды болды.

Сонымен қатар, гидростатикалық тепе-теңдік теңдеуі және әр профиль үшін радиалды координатаның функциясы ретінде қысым профилдері тұрғызылды. Қарастырылып отырған галактикадағы ҚМ үшін күй теңдеуі алынды және дыбыс жылдамдығы есептелінді. Эйнасто, изотермиялық, Буркерт және экспоненциалды сфера профилдерінде тығыздықтың артуымен дыбыс жылдамдығы азаятындығы көрсетілді. Егер ортада дыбыс жылдамдығы азаятын болса, онда ол Әлемнің ірі масштабты құрылымдарының қалыптасуына ықпал ететіні космологиядан белгілі. Сондай-ақ, ҚМ-ның үлестіруіндегі сыну көрсеткіші есептелінді. Сыну көрсеткіші галактиканың ортасына жақындаған сайын тұрақты болатыны, ал гало аймағында азаятындығы көрсетілді. Оның сандық мәні вакуумға қарағанда ҚМ бар галактикада көп болды. Бұл сипаттаманы гравитациялық линзалау эффектісін зерттеу үшін қолдануға болады.

Түйін сөздер: қараңғы материя, айналу қисықтары, күй теңдеуі, дыбыс жылдамдығы, сыну көрсеткіші.

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СВОЙСТВА ТЕМНОЙ МАТЕРИИ В ГАЛАКТИКЕ U5750

Аннотация. Проблема темной материи (ТМ) является одной из фундаментальных и пока нерешенных проблем современной астрофизики, космологии и физики элементарных частиц. В отличие от обычного видимого / барионного вещества, ТМ не участвует в электромагнитных взаимодействиях, но проявляет себя только через гравитацию (и, возможно, также через слабое взаимодействие) и поэтому недоступна для прямых наблюдений. Считается, что в настоящее время ТМ составляет около 26,8% от общей массы всех форм материи во Вселенной. Астрономические наблюдения показывают, что ТМ в основном концентрируется вокруг крупномасштабных космических объектов типа галактик и их кластеров.

В данной работе исследуется распределение ТМ в спиральной галактике U5750. Для исследования характеристик данной галактики рассмотрены известные и широко используемые профили плотности, такие

как изотермический, Буркерта, Наварро-Френка-Уайта (НФУ), Эйнасто и Мура. Кроме того, для сравнения рассмотрен профиль экспоненциальной сферы. Обычно он применяется для изучения внутренних частей галактик, но здесь был приведен в качестве примера. Нахождение численного значения параметров профиля гало темной материи представляет собой определенную проблему, которую, на наш взгляд, можно решить путем исследования динамики галактик. При этом учет темной материи в динамике галактик удачно реализуется в рамках Ньютоновского приближения. Для галактики U5750 изотермический профиль показал самый большой Bayesian Information Criterion (BIC) и самый большой χ^2 . Профили НФУ и Мура расходятся при $r \rightarrow 0$, вызывая проблему каспа (the halo problem). Для исследования уравнения состояния ТМ эти два профиля не подходят, хотя на больших расстояниях они дают плоские кривые вращения. А профили Буркерта, Эйнасто и изотермический лучше всего подходят для анализа уравнения состояния ТМ при $r \rightarrow 0$, так как плотность становится конечной. Из всех выше перечисленных профилей Эйнасто имеет минимальный BIC, а изотермический профиль максимальный BIC.

Помимо этого, было решено уравнение гидростатического равновесия и построены профили давления как функция радиальной координаты для каждого профиля. Объединяя профили плотности с профилями давления, получено уравнение состояния ТМ в рассматриваемой галактике. Рассчитана скорость звука в ТМ и показана, что скорость звука ведет себя не однозначно. Для профилей Эйнасто, Буркерта, изотермического и экспоненциальной сферы скорость звука уменьшается с увеличением плотности. Это уникальная особенность ТМ, которая играет ключевую роль в формировании структуры. Из космологии известно, что если скорость звука в среде уменьшается, то она способствует образованию крупномасштабных структур Вселенной. Кроме того, рассчитан показатель преломления в распределении темной материи. Показатель преломления был постоянным вблизи галактического ядра и уменьшался в области гало. Его значение было несколько больше в галактике, заполненной ТМ, чем в вакууме. Эту характеристику можно использовать для измерения и изучения эффектов гравитационного линзирования.

Ключевые слова: темная материя, кривые вращения, уравнение состояния, скорость звука, показатель преломления

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